Nassau County Interscholastic Mathematics League

Contest # 6 **2002-2003**

No Calculators

Problems 25-26 Time limit: 10 minutes.

25. Carly has 369 markers. Out of these, 130 are black, 100 are blue, and the others are neither blue nor black. In order to complete a drawing, she chooses two markers at random and holds one in each hand. Find the probability that one of the two is black and the other is neither blue nor black. Round your answer to the nearest thousandth.

26. Find all integers n for which the fraction $\frac{n^2+4}{n+5}$ will be an integer.

Problems 27-28 Time limit: 10 minutes.

27. On the planet Skyron, there are three species: (i)Grails always tell the truth: (ii) Grahams, who always lie; and (iii) Gilliams, who never speak first, who (when they do speak) always follow a lie by the previous speaker with the truth, and the truth by a previous speaker with a lie. Three aliens (A, B, and C) make the following statements (in this order):
    - A: B is a Grail;
    - B: A is a Gilliam;
    - C: We are all Grails.

How many of the three are Grails?

28. In how many distinguishable arrangements can the letters of the word BOOKKEEPING be arranged with the first and last letters being the same?

Problems 29-30 Time limit: 10 minutes.

29. How many of the positive integers from 1 through 500 inclusive can be expressed as the difference of the squares of two consecutive positive integers?

30. The diameter of a circle is $\overline{AB}$. Point C is the midpoint of one of the semicircles. Point D is on $\overset{\frown}{AC}$. $\overline{AD}$ and $\overline{DB}$ are drawn. Point E is on $\overline{DB}$ with $\overline{CE} \perp \overline{DB}$. If $\overline{AD}$ has length 5 and $\overline{DE}$ has length 2, find length of $\overline{BE}$.

Answers.

25. 0.266 26. 24,-4, -6, -34

27. 0 28. 272160

29. 249 30. 7